

Decentralized Optimal Control for Connected Automated Vehicles at Intersections Including Left and Right Turns

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Abstract—In earlier work, we addressed the problem of optimally controlling on line connected and automated vehicles crossing two adjacent intersections in an urban area to minimize fuel consumption without any explicit traffic signaling and without considering left and right turns. In this paper, we extend the solution of this problem to account for left and right turns under hard safety constraints. Furthermore, we formulate and solve another optimization problem to minimize a measure of passenger discomfort while the vehicle turns at the intersection, and we investigate the associated tradeoff between minimizing fuel consumption and passenger discomfort.

I. INTRODUCTION

Traffic light signaling is the prevailing method used to control the traffic flow through an intersection. Aside from the infrastructure cost and the need for dynamically controlling green/red cycles, traffic light systems can increase the number of rear-end collisions at the intersection [1]. Serious delays can occur during hours of heavy traffic if the light cycle is not adjusted appropriately. These challenges have motivated research efforts for new approaches capable of providing a smoother traffic flow and more fuel-efficient driving while also improving safety.

In one of the earliest efforts, Dresner and Stone [2] proposed the use of a centralized reservation scheme to control a single intersection of two roads with vehicles traveling with similar speed on a single direction on each road, i.e., no turns are allowed. Since then, several other efforts using reservation scheme have been reported in the literature [3]–[5]. Increasing the throughput of an intersection is one desired goal which can be achieved through the travel time optimization of all vehicles located within a radius from the intersection. Several efforts have focused on minimizing vehicle travel time under collision-avoidance constraints [6]–[9]. Lee and Park [10] proposed an approach based on minimizing the overlap in the position of vehicles inside the intersection rather than their arrival times. A detailed discussion of the research in this area reported in the literature to date can be found in [11].

Connected and automated vehicles (CAVs) provide the most intriguing and promising opportunity for enabling users

to better monitor transportation network conditions to reduce fuel consumption, greenhouse gas emissions, travel delays and improve safety. In earlier work [12], we established a decentralized optimal control framework to address the problem of optimally controlling CAVs crossing two adjacent intersections in an urban area, without any explicit traffic signaling and without considering left and right turns, with the objective of minimizing fuel consumption while achieving maximal throughput with a first-in-first-out requirement on CAVs. To enhance safety awareness in our approach, we required CAVs to have constant speed in the region at the center of each intersection, defined as *Merging Zone* (MZ), which is the area of potential lateral collision of the vehicles. We solved an optimal control problem for each CAV entering a specified *Control Zone* (CZ) for each intersection, which subsequently regulates the acceleration/deceleration of the CAV. The optimal control problem involves hard safety constraints that make it nontrivial to ensure the existence of a feasible solution to this problem. Therefore, we established the conditions under which such feasible solutions exist and showed that they can be enforced through an appropriately designed *Feasibility Enforcement Zone* (FEZ) that precedes the CZ in [13].

To consider left and right turns in this framework, “comfort” becomes of fundamental importance in addition to safety. In this paper, we extend the solution of the problem addressed in [12] to account for left and right turns. Since the speed in the MZ can no longer be constant, we relax this condition and we allow the vehicles to vary their speed inside the MZ. Then, another optimization problem is formulated with the objective of minimizing a measure of passenger discomfort while the vehicle turns. Furthermore, we investigate the associated tradeoff between minimizing fuel consumption and passenger discomfort inside the MZ.

The problem of coordinating CAVs at intersections including left and right turns has been addressed before. Kim and Kumar [14] proposed an approach based on Model Predictive Control towards developing an effective intersection management algorithm that achieves system-wide safety and liveness of intersection-crossing traffic. However, in our approach, the objective is to jointly minimize fuel consumption and passenger discomfort for CAVs crossing an intersection, while safety is a hard constraint.

The paper is organized as follows. In Section II, we review the model in [12] and its generalization in [15]. In Section III, we present the model and the analytical solution of the decentralized optimal control problem inside the CZ for each CAV based on the new collision-avoidance terminal

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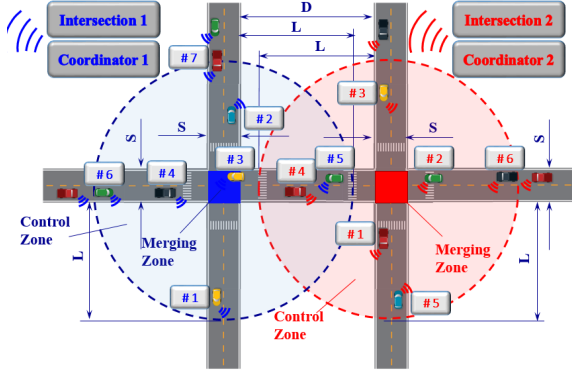


Fig. 1. Connected and automated crossing two adjacent intersections.

conditions. In Section IV, the new optimization problem is formulated for each CAV to address the passenger discomfort inside the MZ and its analytical solution is presented as well. Furthermore, we investigate the associated tradeoff between minimizing fuel consumption and a measure of passenger discomfort. Finally, we present simulation results in Section V, and concluding remarks in Section VI.

II. THE MODEL

We briefly review the model introduced in [12] and [15] where there are two intersections, 1 and 2, located within a distance D (Fig. 1). The region at the center of each intersection, called *Merging Zone* (MZ), is the area of potential lateral CAV collision. Although it is not restrictive, this is taken to be a square of side S . Each intersection has a *Control Zone* (CZ) and a coordinator that can communicate with the CAVs traveling within it. The distance between the entry of the CZ and the entry of the MZ is $L > S$, and it is assumed to be the same for all entry points to a given CZ.

Let $M_z(t) \in \mathbb{N}$ be the cumulative number of CAVs which have entered the CZ and formed a queue by time t , $z = 1, 2$. The way the queue is formed is not restrictive to our analysis in the rest of the paper. When a CAV reaches the CZ of intersection z , the coordinator assigns it an integer value $i = M_z(t) + 1$. If two or more CAVs enter a CZ at the same time, then the corresponding coordinator selects randomly the first one to be assigned the value $M_z(t) + 1$. In the region between the exit point of a MZ and the entry point of the subsequent CZ, the CAVs cruise with the speed they had when they exited that MZ.

For simplicity, we assume that each CAV is governed by a second order dynamics

$$\dot{p}_i = v_i(t), \quad p_i(t_i^0) = 0; \quad \dot{v}_i = u_i(t), \quad v_i(t_i^0) \text{ given} \quad (1)$$

where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$ denote the position, i.e., travel distance since the entry of the CZ, speed and acceleration/deceleration (control input) of each CAV i . The sets \mathcal{P}_i , \mathcal{V}_i and \mathcal{U}_i are complete and totally bounded sets of \mathbb{R} . These dynamics are in force over an interval $[t_i^0, t_i^f]$, where t_i^0 and t_i^f are the times that the vehicle i enters the CZ and exits the MZ of intersection z respectively.

To ensure that the control input and vehicle speed are within a given admissible range, the following constraints are imposed:

$$u_{i,min} \leq u_i(t) \leq u_{i,max}, \quad \text{and} \quad 0 \leq v_{min} \leq v_i(t) \leq v_{max}, \quad \forall t \in [t_i^0, t_i^m], \quad (2)$$

where t_i^m is the time that the vehicle i enters the MZ. To ensure the absence of any rear-end collision throughout the CZ, we impose the *rear-end safety* constraint

$$s_i(t) = p_k(t) - p_i(t) \geq \delta, \quad \forall t \in [t_i^0, t_i^m] \quad (3)$$

where δ is the *minimal safe distance* allowable and k is the CAV physically ahead of i .

The objective of each CAV is to derive an optimal acceleration/deceleration, in terms of fuel consumption, inside the CZ, i.e., over the time interval $[t_i^0, t_i^m]$, while avoiding congestion between the two intersections. In addition, we impose safety constraints to avoid either rear-end collision, or lateral collision inside the MZ. The conditions under which the rear-end collision avoidance constraint does not become active inside the CZ are provided in [13].

III. VEHICLE COORDINATION AND CONTROL

A. Modeling Left and Right Turns

In order to include left and right turns, we impose the following assumptions:

Assumption 3.1: Each vehicle i has proximity sensors and can observe and/or estimate local information that can be shared with other vehicles.

Assumption 3.2: The decision of each vehicle i on whether a turn is to be made at the MZ is known upon its entry in the CZ.

Let d_i denote the decision of vehicle i on whether a turn is to be made at the MZ, where $d_i = 0$ indicates left turn, $d_i = 1$ indicates going straight and $d_i = 2$ indicates right turn.

Assumption 3.3: For each vehicle i , the speed remains constant after the MZ exit for at least a length of δ .

Note that δ is the minimum safety distance allowable in (3). The last assumption is a reasonable consideration since the objective inside the CZ is to minimize the acceleration/deceleration of each CAV in terms of fuel consumption. Thus, there is no compelling reason for the vehicles to accelerate or decelerate right after they exit the MZ and until they enter the CZ of the adjacent intersection (Fig. 1), unless safety is involved.

Left and right turns need special attention in the context of safety while ensuring passenger comfort. We impose the following three *Maximal Allowable Speed* limits inside the MZ: (1) v_L^a for CAVs planning to make left turns, (2) v_R^a for CAVs making right turns, and (3) v^a for CAVs going straight.

The objective for each CAV is to make a safe turn while minimizing a measure of passenger discomfort. There are different ways to define comfort. As a vehicle turns, it is affected simultaneously by two forces: the braking force and

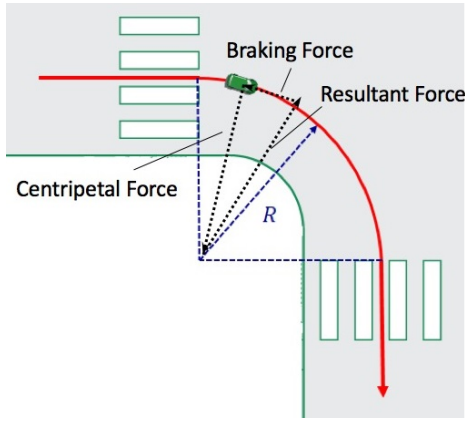


Fig. 2. Vehicle turning process.

the centripetal force (see Fig.2). The directions of the two forces are perpendicular to each other and can be combined into a resultant force. One approach is to keep the magnitude of this resultant force unchanged throughout the turning process. Note that the direction is still changing. Therefore, if the magnitude of the resultant force is unchanged, the vehicle can move in a steady way, and thus the passengers will be less affected by the turn.

B. Vehicle Communication Structure

Since the coordinator is not involved in any decision making on the vehicle control, we can formulate $M_1(t)$ and $M_2(t)$ decentralized problems for intersection 1 and 2 respectively that can be solved on line. When a CAV enters a CZ, $z = 1, 2$, it is assigned a pair (i, j) from the coordinator, where $i = M_z(t) + 1$ is a unique index and $j \in \{1, \dots, 4\}$ is an integer based on a one-to-one mapping from the sets $\{\mathcal{E}_i^z(t), \mathcal{S}_i^z(t), \mathcal{L}_i^z(t), \mathcal{O}_i^z(t)\}$, defined next, onto $\{1, \dots, 4\}$ that indicates the positional relationship between CAVs $i-1$ and i . With respect to CAV i , $i-1$ belongs to one and only one of these subsets defined as follows:

(i) If $j = 1$, then $i-1 \in \mathcal{E}^z(t)$, which contains all vehicles that can cause rear-end collision at the *end* of the MZ with i , e.g., $\mathcal{E}_3^z(t)$ contains vehicle # 2 as it may cause rear-end collision with vehicle # 3 at the end of the MZ (Fig. 3 (a)), and $\mathcal{E}_4^z(t)$ contains vehicle # 3 as it may cause rear-end collision with vehicle # 4 at the end of the MZ (Fig. 3 (a)). Note that this subset does not contain the indices corresponding to vehicles cruising on the same lane and towards the same direction.

(ii) If $j = 2$, then $i-1 \in \mathcal{S}^z(t)$, which contains all vehicles traveling on the same lane that can cause rear-end collision at the *beginning* of the MZ as i , e.g., $\mathcal{S}_3^z(t)$ contains vehicle # 2 as it may cause rear-end collision with vehicle # 3 at the beginning of the MZ (Fig. 3 (b)), and $\mathcal{S}_4^z(t)$ contains vehicle # 3 as it may cause rear-end collision with vehicle # 4 at the beginning of the MZ (Fig. 3 (b)).

(iii) If $j = 3$, then $i-1 \in \mathcal{L}^z(t)$, which contains all vehicles traveling on different lanes and towards different lanes that can cause lateral collision inside the MZ as i , e.g., $\mathcal{L}_3^z(t)$ contains vehicle # 2 as it may cause lateral collision

with vehicle # 3 inside the MZ (Fig. 3 (c)), and $\mathcal{L}_4^z(t)$ contains vehicle # 3 as it may cause lateral collision with vehicle # 4 inside the MZ (Fig. 3 (c)).

(iv) If $j = 4$, then $i-1 \in \mathcal{O}^z(t)$, which contains all vehicles traveling on different lanes and towards different directions that cannot cause lateral collision at the MZ as i , e.g., $\mathcal{O}_3^z(t)$ contains vehicle # 2 since it cannot cause any collision with vehicle # 3 (Fig. 3 (d)), and $\mathcal{O}_4^z(t)$ contains vehicle # 3 since it cannot cause any collision with vehicle # 4 (Fig. 3 (d)).

There is a number of ways to designate the structure of the queue formed by all vehicles inside the CZ. In this paper, we consider a first-in-first-out (FIFO) queue by imposing the following condition:

$$t_i^f \geq t_{i-1}^f, \quad i > 1. \quad (4)$$

Since the speed and the trajectories in the MZ may be different for CAV $i-1$ and i , $t_i^f \geq t_{i-1}^f$ does not imply $t_i^m \geq t_{i-1}^m$. This becomes an issue only when CAV $i-1$ is traveling on the same lane with CAV i . Let k be the index of the CAV physically located ahead of i ($k \leq i-1$ in general). Recall that in earlier work [12] the speed in the MZ was considered to be constant while all CAVs had the same trajectory within the MZ, i.e., S , as left and right turns were not considered and thus, $t_i^m > t_k^m$ can be implicitly ensured by $t_i^f > t_k^f$. As the FIFO queue is defined at t_i^f rather than t_i^m , we simply want to ensure the absence of the rear-end collision at t_i^m . Hence, the following condition is applied:

$$t_i^m > t_k^m, \quad i > k \geq 1. \quad (5)$$

For each CAV i , we define its *information set* $Y_i(t)$, $t \in [t_i^0, t_i^f]$, as

$$Y_i(t) \triangleq \{p_i(t), v_i(t), j, z = 1, 2, s_i(t), t_i^m, t_i^f, d_i\}, \quad (6)$$

where $p_i(t), v_i(t)$ are the traveling distance and speed of CAV i inside the CZ it belongs to, and j indicates the subset to which CAV $i-1$ belongs among $\{\mathcal{E}_i^z(t), \mathcal{S}_i^z(t), \mathcal{L}_i^z(t), \mathcal{O}_i^z(t)\}$, $z = 1, 2$. The fourth element in $Y_i(t)$ is $s_i(t) = p_k(t) - p_i(t)$, the distance between CAV i and CAV k which is immediately ahead of i in the same lane (the index k is made available to i by the coordinator under the Assumption 3.1 that each CAV has proximity sensors and can observe and/or estimate local information that can be shared with other CAVs). t_i^m and t_i^f are the times targeted for CAV i to enter and exit the MZ respectively, whose evaluation are discussed next. The last element d_i , indicates whether i is making left or right turn, or going straight at the MZ, which becomes known once the vehicle enters the CZ (Assumption 3.2). Note that once CAV i enters the CZ, then all information in $Y_i(t)$ becomes available to i .

For safety, comfort and fuel efficiency, it is appropriate for vehicles to make turns at an intersection at low speeds. The speed for which an intersection curve is designed depends on speed limit, the type of intersection, and the traffic volume [16]. Generally, the “desirable time” Δ_i that a vehicle needs to make a turn at an intersection [16] is

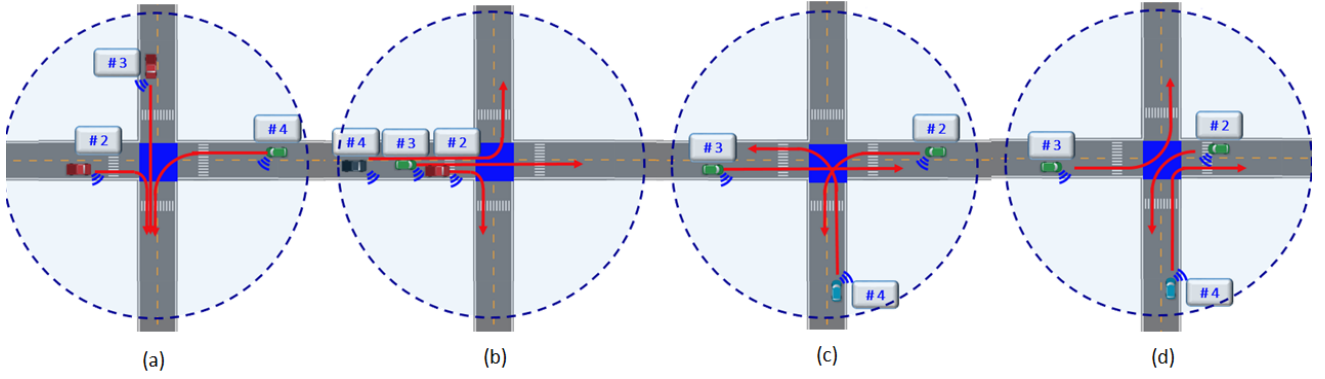


Fig. 3. Illustration of different subsets of $\mathcal{Q}^z(t)$: (a) subset $\mathcal{E}^z(t)$; (b) subset $\mathcal{S}^z(t)$; (c) subset $\mathcal{L}^z(t)$; (d) subset $\mathcal{O}^z(t)$

$$\Delta_i = \begin{cases} \frac{R_i}{\sqrt{15R_i(0.01E+F)}}, & \text{if } d_i = 0, 2, \\ \frac{S}{v_i^a}, & \text{if } d_i = 1, \end{cases} \quad (7)$$

where R_i is the centerline turning radius (see Fig. 2), E is the super-elevation, which is zero in urban conditions, and F is the side friction factor. Therefore, the time t_i^m that the vehicle i enters the MZ is directly related to the time t_i^f that the vehicle exits the MZ through Δ_i :

$$t_i^f = t_i^m + \Delta_i. \quad (8)$$

Note that Δ_i is different for left and right turns since the turning radii R_i for left and right turns are different.

C. Terminal Conditions

We now turn our attention to the terminal conditions, e.g., time for exiting the MZ, speed, and position (known by the geometry of the CZ and MZ), of each vehicle i inside the CZ. The time and speed are designated by the index j as follows.

(i) If $j = 1$, then $i - 1 \in \mathcal{E}_i^z(t)$. In this case, to avoid rear-end collision at the end of the MZ, $i - 1$ and i should maintain a *Minimal Safety Distance* δ , by the time vehicle i exits the MZ. Since for each CAV the speed remains constant after the MZ exit for at least a length δ (Assumption 3.3), we set

$$t_i^f = t_{i-1}^f + \frac{\delta}{v_{i-1}^f} \quad (9)$$

where t_{i-1}^f and t_i^f is the time that vehicle $i - 1$ and i exits the MZ, and v_{i-1}^f is the speed of the vehicle $i - 1$ at the exit of the MZ, which is set as follows:

$$v_{i-1}^f = v_{i-1}^m = \begin{cases} v_L^a, & \text{if } d_{i-1} = 0, \\ v^a, & \text{if } d_{i-1} = 1, \\ v_R^a, & \text{if } d_{i-1} = 2, \end{cases} \quad (10)$$

where v_{i-1}^m is the speed of the vehicle $i - 1$ at the entry of the MZ. In our earlier work [12], we considered a constant speed for the CAVs inside the MZ. To consider left and right turns, the speed can no longer be constant. Furthermore, comfort becomes of fundamental importance in addition to safety. Therefore, we formulate a new optimization problem

to address a measure of passenger discomfort within the MZ. For this problem, (10) are the initial and terminal conditions of the speed for each CAV, and t_i^m is the initial time which can be evaluated according to (8) given t_i^f . Note that v_{i-1}^m in (10) and t_i^m are the terminal conditions for the decentralized optimal control problem in the CZ. While (4) can be satisfied given (9), we need to ensure that (5) also holds since both the speed and the trajectories of CAVs in the MZ may be different. Therefore, t_i^m needs also to be evaluated based on t_k^m in case that vehicle k (i.e., vehicle $i - 1$ denoted as vehicle k when it is the physically located ahead of i), and (5) is violated.

(ii) If $j = 2$, then $i - 1 \in \mathcal{S}_i^z(t)$. In this case, to guarantee the rear-end collision constraint does not become active we set,

$$t_i^m = t_{i-1}^m + \Delta_{i-1}^\delta, \quad (11)$$

where

$$\Delta_{i-1}^\delta = \begin{cases} \frac{\delta}{\sqrt{15R_i(0.01E+F)}}, & \text{if } d_{i-1} = 0, 2, \\ \frac{\delta}{v^a}, & \text{if } d_{i-1} = 1. \end{cases} \quad (12)$$

Δ_{i-1}^δ is the time vehicle $i - 1$ needs to travel a distance δ inside the MZ. The time t_i^f that the vehicle i will be exiting the MZ can be evaluated from (8) while v_i^f is defined as in (10).

Let S_L denote the length of the left turn trajectory and S_R denote the length of the right turn trajectory. If vehicle $i - 1$ makes a left turn and vehicle i makes a right turn, since $S_L > S_R$, $t_i^f < t_{i-1}^f$, and thus (4) does not hold. In that case, we set $t_i^f = t_{i-1}^f$. In this case, t_i^m is evaluated according to (8), which will make t_i^m larger, and thus, (5) still holds.

(iii) If $j = 3$, then $i - 1 \in \mathcal{L}_i^z(t)$. In this case, we constrain the MZ to contain only one CAV so as to avoid lateral collision. The latter constraint is intended to enhance safety awareness, but it could be modified appropriately, if necessary. Therefore, vehicle i will enter the MZ only after vehicle $i - 1$ has exited the MZ, namely

$$t_i^m = t_{i-1}^f. \quad (13)$$

The time t_i^f can be evaluated through (8) and v_i^f is defined as in (10).

(iv) $j = 4$, then $i - 1 \in \mathcal{O}_i^z(t)$. In this case, both $i - 1$ and i can share the MZ at the same time, so we set

$$t_i^m = t_{i-1}^m. \quad (14)$$

The time t_i^f can be evaluated through (8) and v_i^f is defined as in (10).

It follows from (8) through (14) that t_i^f and t_i^m can always be recursively determined from t_{i-1}^m and v_i^m . Similarly, v_i^m depends only on v_{i-1}^m . Although (8) through (14) provide a simple recursive structure for determining t_i^m , the presence of the control and state constraints (2) may prevent these values from being admissible. This may happen by (2) becoming active at some internal point during an optimal trajectory (see [15] for details). In addition, however, there is a global lower bound to t_i^m , which depends on t_i^0 and on whether CAV i can reach v_{max} prior to t_{i-1}^m or not: (i) If CAV i enters the CZ at t_i^0 , accelerates with $u_{i,max}$ until it reaches v_{max} and then cruises at this speed until it leaves the MZ at time t_i^1 , it was shown in [12] that

$$t_i^1 = t_i^0 + \frac{L}{v_{max}} + \frac{(v_{max} - v_i^0)^2}{2u_{i,max}v_{max}}. \quad (15)$$

(ii) If CAV i accelerates with $u_{i,max}$ but reaches the MZ at t_i^m with speed $v_i^m < v_{max}$, it was shown in [12] that

$$t_i^2 = t_i^0 + \frac{v_i(t_i^m) - v_i^0}{u_{i,max}}, \quad (16)$$

where $v_i(t_i^m) = \sqrt{2Lu_{i,max} + (v_i^0)^2}$. Thus,

$$t_i^c = t_i^1 \mathbb{1}_{v_i^m=v_{max}} + t_i^2 (1 - \mathbb{1}_{v_i^m=v_{max}})$$

is a lower bound of t_i^m regardless of the solution of the problem.

D. Decentralized Control Problem Formulation and Analytical Solution

Recall that at time t , the values of t_{i-1}^f , t_{i-1}^m , v_{i-1}^f , $j = 1, \dots, 4$, $z = 1, 2$, d_{i-1} are available to CAV i through its information set in (6). This is necessary for i to compute t_i^f and t_i^m appropriately and satisfy (4) and (5). The following result is established in [12] to formally assert the iterative structure of the sequence of decentralized optimal control problems:

Lemma 1: The decentralized communication structure aims for each CAV i to solve an optimal control problem for $t \in [t_i^0, t_i^m]$ the solution of which depends only on the solution of CAV $i-1$ [12].

The decentralized optimal control problem for each CAV approaching either intersection is formulated so as to minimize the L^2 -norm of its control input (acceleration/deceleration). There is a monotonic relationship between fuel consumption for each CAV i , and its control input u_i [17]. Therefore, we formulate the following problem for each

i :

$$\min_{u_i \in U_i} \frac{1}{2} \int_{t_i^0}^{t_i^m} K_i \cdot u_i^2 dt$$

$$\text{subject to : (1), (2), } t_i^m, p_i(t_i^0) = 0, p_i(t_i^m) = L, \quad (17)$$

$$z = 1, 2, \text{ and given } t_i^0, v_i(t_i^0),$$

where K_i is a factor to capture CAV diversity (for simplicity we set $K_i = 1$ for the rest of this paper). Note that this formulation does not include the safety constraint (3).

An analytical solution of problem (17) may be obtained through a Hamiltonian analysis. The presence of constraints (2) and (3) complicates this analysis. The complete solution including any constraint in (2) becoming active is given in [15]. Assuming that all constraints are satisfied upon entering the CZ and that they remain inactive throughout $[t_i^0, t_i^m]$, a complete solution was derived in [17] and [18] for highway on-ramps, and in [12] for two adjacent intersections. The solution to (17) differs from what we have derived in [12] presumably in the values of the coefficients instead of the structure, as the terminal conditions from (8) through (14) consider left and right turns. The optimal control input (acceleration/deceleration) over $t \in [t_i^0, t_i^m]$ is given by

$$u_i^*(t) = a_i t + b_i \quad (18)$$

where a_i and b_i are constants. Using (18) in the CAV dynamics (1) we also obtain the optimal speed and position:

$$v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i \quad (19)$$

$$p_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i, \quad (20)$$

where c_i and d_i are constants of integration. The constants a_i , b_i , c_i , d_i can be computed by using the given initial and final conditions. The analytical solution (18) is only valid as long as all initial conditions satisfy (2) and (3) and none of these constraints becomes active in $[t_i^0, t_i^m]$. Otherwise, the solution needs to be modified as described in [15]. Recall that the constraint (3) is not included in (17) and it is a much more challenging matter. To address this, we derive the conditions under which the CAV's state maintains feasibility in terms of satisfying (3) over $[t_i^0, t_i^m]$ in [13].

IV. JOINT MINIMIZATION OF PASSENGER DISCOMFORT AND FUEL CONSUMPTION IN THE MZ

A. Passenger Discomfort

It is reported in [19] that the comfort of the passengers in transportation can be quantified as a function of jerk, which is the derivative of acceleration with respect to time, i.e., $J_i(t) = \dot{u}_i(t)$. One approach to minimize passenger discomfort is to keep the magnitude of the resultant force, which consists of the centripetal force and the braking force, unchanged. Note that both the magnitude of the centripetal force and the angle between the centripetal and braking forces do not change while the vehicle makes a turn. Hence, the following optimization problem is formulated with the objective of minimizing the L^2 -norm of jerk for each vehicle

i , where the acceleration/deceleration $u_i(t)$ is the control input:

$$\begin{aligned} \min_{u_i} \quad & \frac{1}{2} \int_{t_i^m}^{t_i^f} J_i^2 dt \\ \text{subject to : } & (1), J_i(t) = \dot{u}_i(t), \\ & u_i(t_i^m), u_i(t_i^f), v_i^m, v_i^f, p_i(t_i^m), p_i(t_i^f), \text{ given } t_i^m, t_i^f, \end{aligned} \quad (21)$$

The analytical solution of problem (21) has been obtained in [20] using Hamiltonian analysis and considering the jerk as the control input. However, here we control jerk indirectly through the acceleration/deceleration, so the analytical, close-form solution is

$$u_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i, \quad (22)$$

$$v_i^*(t) = \frac{1}{24} a_i t^4 + \frac{1}{6} b_i t^3 + \frac{1}{2} c_i t^2 + d_i t + e_i, \quad (23)$$

$$p_i^*(t) = \frac{1}{120} a_i t^5 + \frac{1}{24} b_i t^4 + \frac{1}{6} c_i t^3 + \frac{1}{2} d_i t^2 + e_i t + f_i, \quad (24)$$

where a_i , b_i , c_i , d_i , e_i and f_i are constants of integration, which can be computed by using the given initial and final conditions at t_i^m and t_i^f . Therefore, (22) is the analytical optimal control input corresponding to (21) that will yield the minimum L^2 -norm of jerk for vehicle i inside the MZ.

B. Tradeoff between Fuel Consumption and Passenger Discomfort

To investigate the tradeoff between fuel consumption and passenger discomfort, we formulate a joint objective expressed as a convex combination of acceleration/deceleration and jerk. The optimization problem is formulated as follows:

$$\min_{u_i} \quad \frac{1}{2} \int_{t_i^m}^{t_i^f} (w \cdot q_1 \cdot u_i^2 + (1-w) \cdot q_2 \cdot J_i^2) dt \quad (25)$$

$$\text{subject to : } (1), J_i(t) = \dot{u}_i(t)$$

$$0 \leq w \leq 1,$$

$$u_i(t_i^m), u_i(t_i^f), v_i^m, v_i^f, p_i(t_i^m) \text{ and } p_i(t_i^f), \text{ given } t_i^m, t_i^f.$$

where q_1 , q_2 are normalization factors which are selected so that $q_1 \cdot u_i^2 \in [0, 1]$ and $q_2 \cdot J_i^2 \in [0, 1]$. The Hamiltonian function of (25) becomes

$$\begin{aligned} H_i(t, x(t), j(t)) = & w \cdot q_1 \cdot \frac{1}{2} u_i^2 + (1-w) \cdot q_2 \cdot \frac{1}{2} J_i^2 \\ & + \lambda_i^p \cdot v_i + \lambda_i^v \cdot u_i + \lambda_i^u \cdot J_i, \end{aligned}$$

where λ_i^p , λ_i^v and λ_i^u are the co-state components. Given the necessary conditions for optimality, we have the following second-order ordinary differential equation,

$$(1-w) \cdot q_2 \cdot \ddot{v} - w \cdot q_1 \cdot v + \frac{1}{2} a_i t^2 + b_i t + c_i = 0$$

from which, we can derive the optimal solution as

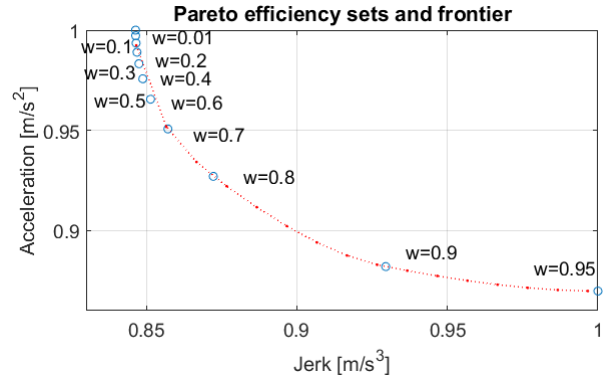


Fig. 4. Pareto efficiency sets and frontier corresponding to different combinations of fuel consumption and passenger discomfort in the MZ.

$$J_i^*(t) = \frac{a_i}{w q_1} + e_i A_1^3 e^{A_1 t} + f_i A_2^3 e^{A_2 t} \quad (26)$$

$$u_i^*(t) = \frac{1}{w q_1} (a_i t + b_i) + e_i A_1^2 e^{A_1 t} + f_i A_2^2 e^{A_2 t} \quad (27)$$

$$\begin{aligned} v_i^*(t) = & \frac{1}{w q_1} \left(\frac{1}{2} a_i t^2 + b_i t + c_i + \frac{a_i (1-w) q_2}{w q_1} \right) \\ & + e_i A_1 e^{A_1 t} + f_i A_2 e^{A_2 t} \end{aligned} \quad (28)$$

$$\begin{aligned} p_i^*(t) = & \frac{1}{w q_1} \left(\frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + \frac{a_i (1-w) q_2}{w q_1} t + d_i \right) \\ & + e_i e^{A_1 t} + f_i e^{A_2 t} \end{aligned} \quad (29)$$

where

$$A_1 = \sqrt{\frac{w q_1}{(1-w) q_2}}, \quad A_2 = -\sqrt{\frac{w q_1}{(1-w) q_2}}.$$

The constants a_i , b_i , c_i , d_i , e_i and f_i can be computed using the initial and final conditions at t_i^m and t_i^f . Note that since $0 \leq w \leq 1$, the optimal solution is only valid when $w \neq 1$ and $w \neq 0$. When $w = 0$, the problem becomes the original formulation (21) with the objective of minimizing jerk only. When $w = 1$, the problem becomes the same as the one we formulate in the CZ, which minimizes the fuel consumption in the MZ:

$$\min_{u_i} \quad \frac{1}{2} \int_{t_i^m}^{t_i^f} u_i^2 dt \quad (30)$$

$$\text{subject to : } (1), v_i^m, v_i^f, p_i(t_i^m), p_i(t_i^f), \text{ given } t_i^m, t_i^f.$$

The optimal solution to (25) varies as the weight w changes. To illustrate the tradeoff between passenger discomfort and fuel consumption, we examine a range of cases with different weights and produce the Pareto sets. The parameters used are listed in Sec. V except for the initial and terminal acceleration which are set to 0. In a Pareto efficiency allocation, no objective can be made better off without making at least one objective worse. By yielding all of the optimal solutions to (25) while varying the weight w , we can derive the Pareto sets and the Pareto frontier

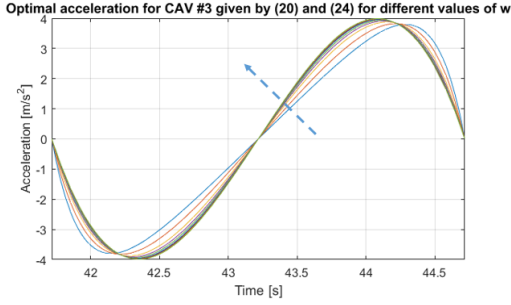


Fig. 5. Optimal acceleration trajectory for CAV #3 given (21) and (25) with different value of w .

corresponding to different combinations of fuel consumption and passenger discomfort as shown in Fig. 4. Apparently, there is a tradeoff between fuel consumption and a measure of comfort while the vehicle turns at the intersection.

Note that as the weight w goes to 0, (25) should behave similar to (21). Hence, the limit of the optimal solution to (25) as the weight $w \rightarrow 0$ should be the optimal solution to (21). In Fig. 5, we can observe that as the weight w decreases from 0.95 to 0.01 along the direction of the blue arrow, the optimal acceleration trajectory given by (25) is asymptotically approaching the one derived from (21).

V. SIMULATION EXAMPLES

The proposed decentralized optimal control framework incorporating turns is illustrated through simulation in MATLAB. For each direction, only one lane is considered. The parameters used are: $L = 400$ m, $S = 30$ m, $S_L = \frac{3}{8}\pi S$, $S_R = \frac{1}{8}\pi S$, $\delta = 10$ m, $v_L^a = 8$ m/s, $v_R^a = 6$ m/s, $v^a = 10$ m/s and $\Delta_i = 5, 3, 3$ s for left turn, going straight and right turn respectively. CAVs arrive at the CZ based on a random arrival process. Here, we assume a Poisson arrival process with rate $\lambda = 1$ and the speeds are uniformly distributed over [10, 12].

We first consider the case where only the L^2 -norm of jerk is optimized over the MZ. The initial and terminal conditions of time and speed are defined from (8) through (14). Observe that $p_i(t_i^m) = L$ and $p_i(t_i^f) = L + S$ if i is going straight ($p_i(t_i^f) = L + S_L$ for left turn and $p_i(t_i^f) = L + S_R$ for right turn). The two additional conditions needed for acceleration/deceleration are set as follows: (a) the initial acceleration for (21) is set to the terminal value derived from the optimal control problem in the CZ (17), under which the acceleration/deceleration is continuous at t_i^m and the smoothness can be achieved in terms of avoiding jumps of acceleration; (b) the terminal acceleration is set as zero. The position trajectories of the first 30 CAVs in the MZ are shown in Fig. 6. CAVs are separated into two groups: CAVs shown above zero are driving from east or west, and those below zero are driving from north or south, with labels indicating the position of the vehicles in the FIFO queue and the driving direction. These figures include different instances from each of Cases 1), 2), 3) or 4) in Sec. III-B regarding the value of t_i^f . For example, CAV #11 is

assigned $t_{11}^m = t_{10}^f$, which corresponds to Case 3), CAV #23 is assigned $t_{23}^m = t_{22}^m$, which corresponds to Case 4), CAV #27 is assigned $t_{27}^f = t_{26}^f + \frac{\delta}{26}$, which corresponds to Case 1), whereas CAV #13 is assigned $t_{13}^m = t_{12}^m + \Delta_{12}^\delta$, which corresponds to Case 2).

To demonstrate the effectiveness of our optimal control in the MZ and compare formulations with different objectives, we examine three cases: (a) the objective is to minimize the L^2 -norm of acceleration/deceleration only (30); (b) the objective is to minimize the L^2 -norm of jerk only (21); (c) the objective is to minimize the weighted sum of L^2 -norm of acceleration/deceleration and L^2 -norm of jerk (25), where $w = 0.95$. Note that all terms should be normalized into a uniform, dimensionless scale for multi-objective optimization. The acceleration and jerk profiles of the first 30 CAVs are shown in Fig. 7. Note that the optimal solution depends on how we set the initial and terminal acceleration/deceleration.

VI. CONCLUSIONS

Earlier work [12] has established a decentralized optimal control framework for optimally controlling CAVs crossing two adjacent intersections in an urban area. In this paper, we extended the solution of this problem to account for left and right turns. In addition, we formulated and solved another optimization problem to minimize a measure of passenger discomfort while the vehicle turns at the intersection, and investigated the associated tradeoff between minimizing fuel consumption and passenger discomfort. The optimal solution including turns do not require any additional computational time than what is required by the solution in [12] since the terminal conditions are basically determined based on another set of collision-avoidance constraints, which can still enable online implementation. Future research should investigate the implications of having information with errors and/or delays to the system behavior.

REFERENCES

- [1] <http://www.dot.state.oh.us/districts/D01/PlanningPrograms/trafficstudies/Pages/TrafficSignals.aspx>.
- [2] K. Dresner and P. Stone, "Multiagent traffic management: a reservation-based intersection control mechanism," in *Proceedings of the Third International Joint Conference on Autonomous Agents and Multiagents Systems*, 2004, pp. 530–537.
- [3] —, "A Multiagent Approach to Autonomous Intersection Management," *Journal of Artificial Intelligence Research*, vol. 31, pp. 591–653, 2008.
- [4] A. de La Fortelle, "Analysis of reservation algorithms for cooperative planning at intersections," *13th International IEEE Conference on Intelligent Transportation Systems*, pp. 445–449, Sept. 2010.
- [5] S. Huang, A. Sadek, and Y. Zhao, "Assessing the Mobility and Environmental Benefits of Reservation-Based Intelligent Intersections Using an Integrated Simulator," *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, no. 3, pp. 1201,1214, 2012.
- [6] L. Li and F.-Y. Wang, "Cooperative Driving at Blind Crossings Using Intervehicle Communication," *IEEE Transactions in Vehicular Technology*, vol. 55, no. 6, pp. 1712,1724, 2006.
- [7] F. Yan, M. Dridi, and A. El Moudni, "Autonomous vehicle sequencing algorithm at isolated intersections," *2009 12th International IEEE Conference on Intelligent Transportation Systems*, pp. 1–6, 2009.
- [8] I. H. Zohdy, R. K. Kamalanathsharma, and H. Rakha, "Intersection management for autonomous vehicles using iCACC," *2012 15th International IEEE Conference on Intelligent Transportation Systems*, pp. 1109–1114, 2012.

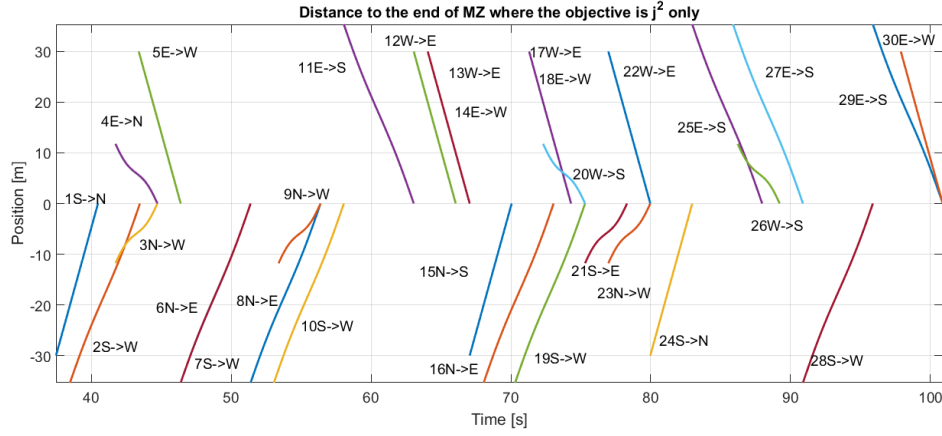


Fig. 6. Distance to the end of MZ of the first 30 CAVs in the MZ.

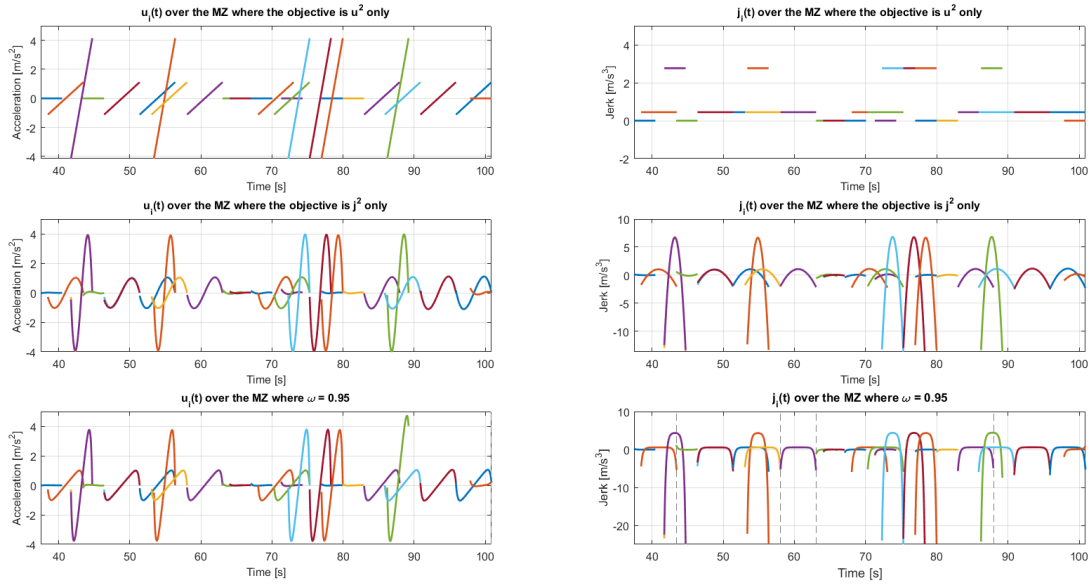


Fig. 7. Acceleration/deceleration $u_i(t)$ and jerk $J_i(t)$ trajectories for the cases with different objectives: (a) minimize fuel consumption only; (b) minimize passenger discomfort only; (c) minimize a weighted sum of fuel consumption and passenger discomfort where $w = 0.95$

- [9] F. Zhu and S. V. Ukkusuri, "A linear programming formulation for autonomous intersection control within a dynamic traffic assignment and connected vehicle environment," *Transportation Research Part C: Emerging Technologies*, 2015.
- [10] J. Lee and B. Park, "Development and Evaluation of a Cooperative Vehicle Intersection Control Algorithm Under the Connected Vehicles Environment," *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, no. 1, pp. 81–90, 2012.
- [11] J. Rios-Torres and A. A. Malikopoulos, "A Survey on the Coordination of Connected and Automated Vehicles at Intersections and Merging at Highway On-Ramps," *IEEE Transactions on Intelligent Transportation Systems*, 2016 (forthcoming).
- [12] Y. Zhang, A. A. Malikopoulos, and C. G. Cassandras, "Optimal control and coordination of connected and automated vehicles at urban traffic intersections," in *Proceedings of the American Control Conference*, 2016, pp. 6227–6232.
- [13] Y. Zhang, C. G. Cassandras, and A. A. Malikopoulos, "Optimal control of connected automated vehicles at urban traffic intersections: A feasibility enforcement analysis," in *Proceedings of the 2017 American Control Conference*, to appear.
- [14] K.-D. Kim and P. R. Kumar, "An mpc-based approach to provable system-wide safety and liveness of autonomous ground traffic," *IEEE Transactions on Automatic Control*, vol. 59, no. 12, pp. 3341–3356, 2014.
- [15] A. A. Malikopoulos, C. G. Cassandras, and Y. Zhang, "A decentralized optimal control framework for connected automated vehicles at urban intersections," *arXiv:1602.03786*, 2016.
- [16] A. Aashto, "Policy on geometric design of highways and streets," *American Association of State Highway and Transportation Officials*, Washington, DC, vol. 1, no. 990, p. 158, 2001.
- [17] J. Rios-Torres, A. A. Malikopoulos, and P. Pisu, "Online Optimal Control of Connected Vehicles for Efficient Traffic Flow at Merging Roads," in *2015 IEEE 18th International Conference on Intelligent Transportation Systems*, Canary Islands, Spain, September 15–18, 2015.
- [18] J. Rios-Torres and A. A. Malikopoulos, "Automated and Cooperative Vehicle Merging at Highway On-Ramps," *IEEE Transactions on Intelligent Transportation Systems*, 2016 (forthcoming).
- [19] N. Hogan, "Adaptive control of mechanical impedance by coactivation of antagonist muscles," *IEEE Transactions on Automatic Control*, vol. 29, no. 8, pp. 681–690, 1984.
- [20] I. A. Ntousakis, I. K. Nikolos, and M. Papageorgiou, "Optimal vehicle trajectory planning in the context of cooperative merging on highways," *Transportation Research Part C: Emerging Technologies*, vol. 71, pp. 464–488, 2016.